PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 45

Total Marks : 31

Max. Time : 32 min.

Topics : Circular Motion, Center of Mass, Newton's Law of Motion, Work , Power and Energy

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 to Q.6	(3 marks, 3 min.)	[18, 18]
Subjective Questions ('–1' negative marking) Q.7	(4 marks, 5 min.)	[4, 5]
Comprehension ('–1' negative marking) Q.8 to Q.10	(3 marks, 3 min.)	[9, 9]

1. A circular curve of a highway is designed for traffic moving at 72 km/h. If the radius of the curved path is 100 m, the correct angle of banking of the road should be given by :

(A) $\tan^{-1}\frac{2}{\ }$ (B) $\tan^{-1}\frac{3}{\ }$ (C) $\tan^{-1}\frac{2}{5}$ (D) $\tan^{-1}\frac{1}{\ }$

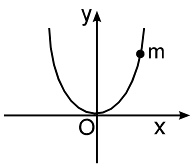
2. Two semicircular rings of linear mass densities λ and 2λ and of radius 'R' each are joined to form a complete ring. The distance of the center of the mass of complete ring from its geometrical centre is :

(A) $\frac{3R}{8\pi}$ (B) $\frac{2R}{3\pi}$ (C) $\frac{3R}{4\pi}$ (D) none of these

3. The centre of mass of a non uniform rod of length L whose mass per unit length λ varies as $\lambda = \frac{k.x}{\ }$ where k is a constant & x is the distance of any point on rod from its one end, is (from the same end)

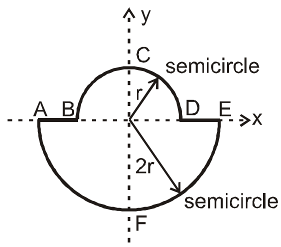
(A) — L (B) L (C) — (D) —

4. A bead of mass m is located on a parabolic wire with its axis vertical and vertex at the origin as shown in figure and whose equation is $x^2 = 4ay$. The wire frame is fixed in vertical plane and the bead can slide on it without friction. The bead is released from the point y = 4a on the wire frame from rest. The tangential acceleration of the bead when it reaches the position given by y = a is :



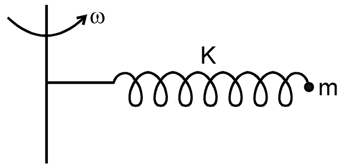
(A) $\frac{g}{2}$ (B) $\frac{\sqrt{3}g}{2}$ (C) $\frac{g}{\sqrt{2}}$ (D) $\frac{g}{\sqrt{5}}$

5. A uniform thin rod is bent in the form of closed loop ABCDEFA as shown in the figure. The y-coordinate of the centre of mass of the system is



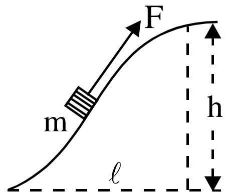
(A) $\frac{2r}{\pi}$ (B) $-\frac{6r}{3\pi+2}$ (C) $-\frac{2r}{\pi}$ (D) Zero

6. A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length ℓ. The system is rotated about the other end of the spring with an angular speed ω ($\omega < \sqrt{\frac{k}{m}}$) in gravity free space. The increase in length of the spring is :



(A) $\frac{m\omega^2\ell}{k}$ (B) $\frac{m\omega^2\ell}{k-m\omega^2}$ (C) $\frac{m\omega^2\ell}{k+m\omega^2}$ (D) none of these

7. A body of mass m was slowly hauled up the hill as shown in figure by a force F which at each point was directed along a tangent to the trajectory. Find the work performed by this force, if the height of the hill is h, the length of its base ℓ, and the coefficient of friction k.



COMPREHENSION

One end of massless inextensible string of length ℓ is fixed and other end is tied to a small ball of mass m. The ball is performing a circular motion in vertical plane. At the lowest position, speed of ball is $\sqrt{20g\ell}$. Neglect any other forces on the ball except tension and gravitational force. Acceleration due to gravity is g.

8. Motion of ball is in nature of
(A) circular motion with constant speed
(B) circular motion with variable speed
(C) circular motion with constant angular acceleration about centre of the circle.
(D) none of these

9. At the highest position of ball, tangential acceleration of ball is -
(A) 0 (B) g (C) 5 g (D) 16 g

10. During circular motion, minimum value of tension in the string -
(A) zero (B) mg (C) 10 mg (D) 15 mg

Answers Key

DPP NO. - 45

1. (C) **2.** (B) **3.** (A)
4. (C) **5.** (B) **6.** (B)
7. A = mg (h + kℓ) **8.** (B)
9. (A) **10.** (D)

Hint & Solutions

DPP NO. - 45

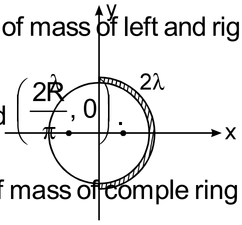
1. $V = \sqrt{gR\tan\theta} \Rightarrow (20)^2 = 10 \times 100 \times \tan\theta$

$\Rightarrow \tan\theta = \frac{4}{10} = \frac{2}{5}$

$\Rightarrow \theta = \tan^{-1}(2/5)$ **Ans: None**

2. Let the two half rings be placed in left and right of y-axis with centre as shown in figure.

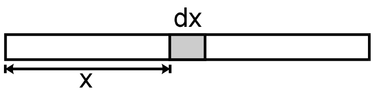
Then the coordinate of centre of mass of left and right half rings are $\left(-\frac{2R}{\pi}, 0\right)$ and $\left(\frac{2R}{\pi}, 0\right)$.



$\therefore$ x-coordinates of centre of mass of comple ring is

$$\frac{m\left(-\frac{2R}{\pi}\right) + 2m\left(\frac{2R}{\pi}\right)}{3m} = \frac{2R}{3\pi}$$

3.

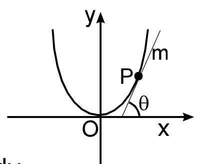


$$\therefore \; x_{cm} = \frac{\int_0^L \frac{K}{L}x^2 dx . x}{\int_0^L \frac{K}{L}x^2 dx} = \frac{\left.\frac{x^4}{4}\right|_0^L}{\left.\frac{x^3}{3}\right|_0^L}$$

$$= \frac{3}{4}L$$

4. $x^2 = 4ay$ Differentiating w.r.t. y, we get

$\frac{dy}{dx} = \frac{x}{2a}$



$\therefore$ At (2a, a), $\frac{dy}{dx} = 1$

$\Rightarrow$ hence $\theta = 45°$

the component of weight along tangential direction is mg sin θ.

hence tangential acceleration is g sin θ = $\surd$

5. The centre of mass of semicircular ring is at a distance $\frac{2r}{\pi}$ from its centre. (Let λ = mass/length)

$$\therefore Y_{cm} = \frac{\lambda \pi r \times \frac{2r}{\pi} - \lambda \times 2\pi r \times \frac{4r}{\pi}}{\lambda \pi r + \lambda r + \lambda r + \lambda \times 2\pi r} = -\frac{6r}{3\pi + 2}$$

6.

$kx = m\omega^2 \ell + m\omega^2 x$

$(k - m\omega^2)\, x = m\omega^2 \ell$

$x = \frac{m\omega^2 \ell}{k - m\omega^2}$ **Ans. (B)**

7. For slowly havled $\Delta K = 0$

$W_F + W_g + W_f = \Delta K$

$W_g = -mgh$

$W_f = -mgk\ell$

$W_F = mgh + mgk\ell = mg\ell\,(h + k\ell)$.

8. As speed of ball is variable, so motion is non uniform circular motion.

9. At the highest position of ball, tangential acceleration of ball is zero,

10. Tension in the string is minimum when ball is at the highest position. By conservation of energy $\frac{1}{2}mv^2$ +

$mg\,(2\ell) = \frac{1}{2}m(20\,g\ell)$

$v^2 = 16\,g\ell$ where v is the velocity of ball at the highest point.

So $T + mg = \frac{mv^2}{\ell}$

$T = \frac{m16g\ell}{\ell} - mg = 15\,mg$